TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 266

AIRFOIL LIFT WITH CHANGING ANGLE OF ATTACK

By Elliott G. Reid
Langley Memorial Aeronautical Laboratory

Washington
September, 1927

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL NOTE NO. 266.

AIRFOIL LIFT WITH CHANGING ANGLE OF ATTACK.

By Elliott G. Reid.

Summary

Tests have been made in the atmospheric wind tunnel of the National Advisory Committee for Aeronautics to determine the effects of pitching oscillations upon the lift of an airfoil.

It has been found that the lift of an airfoil, while pitching, is usually less than that which would exist at the same angle of attack in the stationary condition, although exceptions may occur when the lift is small or if the angle of attack is being rapidly reduced.

It is also shown that the behavior of a pitching airfoil may be qualitatively explained on the basis of accepted aerodynamic theory.

Introduction

As the science of aerodynamics is, as yet, in a state of development, rather than in one of refinement, it is natural that the steady motion of wings through the air has been studied extensively while the essentials of the accelerated motions remain practically unknown. The necessity for attacking the latter problem has been felt for some time; the necessity of investigating the forces which act upon the wings and tail surfaces of modern airplanes during the rapid maneuvers of which they are capable has focused attention upon this field of aerodynamics. Other examples of the unsteady motions of airfoils are found in the autogyro, the helicopter in horizontal or gliding flight, the feathering blade propeller and the flutter of airplane wings.

All of the examples just mentioned give rise to the same questions: (a) Are the instantaneous air forces which act upon an airfoil while its angle of attack is changing equal to those experienced by it while in rectilinear motion at the same angles of attack? (b) If the answer to (a) is negative, what differences exist? The study of this fundamental problem was suggested by Dr. Munk, in whose paper (Reference 1) will be found a theoretical treatment (for the wing of infinite aspect ratio) of the questions stated above.

The experiments which were made in the atmospheric wind tunnel at Langley Memorial Aeronautical Laboratory, as the result of this suggestion, were completed during 1924. An analysis made by the writer at that time led only to conclusions of an indefinite nature. In a recent attempt to formulate the theory of the action of a pitching airfoil, the fallacy of the first analysis was discovered; the elements of the theory are presented in the discussion in support of the results of the tests.

Tests

To try to make direct measurements of the forces acting upon a rotating airfoil was considered impracticable, particularly because it had been decided to make the airfoil rotate about an axis well forward of its leading edge, thus involving displacement as well as simple rotation. As the method used to determine the forces was somewhat unusual, it will be described in detail.

The equation

$$Q = \frac{I\alpha}{g} \qquad (1)$$

expresses the principle of mechanics which was utilized in determining the air forces without resorting to direct measurement. To express this basic theorem literally: the application of the torque Q, to a body having a moment of inertia, I (force units) about the axis of rotation produces an angular acceleration α.

The testing apparatus is diagrammatically illustrated in Figure 1; Figures 2 and 3 are photographs of the tunnel installation. The arm on which the symmetrical airfoil is mounted is free to rotate about the axis a, (a heavy piano wire) and its motion is transmitted to the recording mechanism through the parallel vertical wires b. The shaft c, which is actuated by the motion of the airfoil, carries the recording cylinder d. By moving a pencil along the cylinder in a direction parallel to that of the axis and at a uniform speed, the angular displacement of the airfoil is recorded on a sheet of paper which is wrapped around the cylinder.

The making of a test consisted in displacing the airfoil to a large angle of attack, releasing it and recording the ensuing heavily damped oscillation which resulted from the action of air and inertia forces.

To obtain the forces or, to be exact, the torques, acting on the airfoil, the data were reduced as follows:

A record was measured and plotted (radians vs. seconds) to a large scale. Arithmetical differentiation of this curve gave the coordinates of the corresponding angular velocity vs. time curve. The point values were faired to eliminate the inaccuracies of differentiation and the same process was then applied again; the resulting curve represented angular acceleration vs. time. As the moment of inertia of the oscillating system was known, instantaneous values of the torque produced by the action of the air forces could be obtained by the application of equation (1).

Two other curves were obtained for purposes of analysis. They represent the variations of angle of attack and rate of change of angle of attack with time. In explanation of this procedure, the reader is reminded that the angle between the chord of the wing and the wind direction is not the angle of attack. As the wing experiences displacement, as well as rotation, the angle of attack is greater or less than the geometric angle by an amount which depends upon the sign and magnitude of the tangential velocity. The actual angle of attack was computed by vectorially compounding the velocity components for points distributed over the entire oscillation. Arithmetical differentiation of the curve plotted from these data gave a curve of rate of change of angle of attack (or "aerodynamic angular velocity") vs. time. The point for which the tangential velocities were calculated was one located at one-third chord behind the leading edge of the airfoil. This point was chosen as an approximation of the position of the center of pressure during oscillation; it is believed that this is a reasonably good estimate because the rotation of the symmetrical gives it effective camber.

The plan form and profile of the airfoil used are illustrated in Figure 4. The distance between the axis of rotation and the leading edge of the airfoil was 0.61 m (2 ft.). The air speed was 20 m/s (65.6 ft./sec.).

Supplementary tests were made to determine the moment of inertia of the oscillating system and the torques produced by the air forces at various angles of attack with the wing stationary. The first was accomplished by finding the acceleration of the oscillating system under the action of a known torque; I was found to be 0.12925 kg m². The second was done by restraining the otherwise free system at various angles of attack by means of a spring balance.

With regard to the reduction of data, it is mentioned that the effects of the apparent camber created by the rotation and of the contribution of the tangential velocity to the resultant velocity upon the angle of attack and the air forces, respectively, have been disregarded as both were found to be of negligible magnitude.

Results

The results of a single oscillation test are presented in Figures 5-8. Figure 5 is a reproduction, in actual size, of the oscillation record. In Figure 6 are curves of θ, the angular displacement; ω, the angular velocity; and α, the angular acceleration, all plotted against T the time, with the radian as the angular unit. The variations of θ', the actual angle of attack in radians (and degrees), and of ω', the "aerodynamic angular velocity" in radians per second with time are illustrated by Figure 7. The final results of the tests are presented in Figures 8a-8f; there are shown (a) torque vs. angle of attack for the stationary condition, (b) torque vs. angle of attack for $\omega' = 0.0$, 0.25, 0.50, and 0.75 rad./sec. and (c) a composite of (a) and (b). Only that portion of the oscillation curve corresponding to time values greater than 1.4 seconds has been used for the derivation of the values of θ', ω' and the final results of Figures 3a-8f. This was done because it appears that the airfoil swung beyond the angle of maximum lift at the peaks of the first two arches of the curve.

It will be noticed that the curves occur in pairs in those plots of Figures 8a-8f, for which ω' is finite and that no negative values of θ' are shown. As a matter of fact, the data points represent both positive and negative angles of attack; but all have been plotted in the same quadrant for the purpose of more clearly defining the curves which would, as the airfoil was symmetrical, be double mirror images of each other in the first and third quadrants. Such plotting is also advantageous because it will be seen later that we are only interested in whether θ' is numerically increasing or decreasing.

A word of explanation must be offered for the use of the unconventional symbols: The notation follows the conventions of mathematics and mechanics and the use of θ, ω and α seems less confusing than to use α, $\dot{\alpha}$ and $\ddot{\alpha}$ or α, $\frac{d\alpha}{dt}$ and $\frac{d^2\alpha}{dt^2}$ to express the same quantities, one of which latter forms would be necessary if the conventional aerodynamic nomenclature had to be followed.

Discussion

Before entering upon a critical examination of the results, it would be well to consider the significance of the quantity Q. The torque about the axis of rotation is equal to the product of the normal component of the resultant air force and the distance between its point of application (the center of pressure) and the axis. As the lift is obtained by multiplication of the normal force by the cosine of the angle of attack and as we are concerned only with small values of the latter, the lift at any angle of attack is almost exactly proportional to the corresponding normal force. It is known that the center of pressure travel on thin airfoils of small camber is very minute and when such travel is compared with the distance from the axis of rotation to the center of pressure (approximately 4-1/3 chords in this case) it is found to be of negligible importance. Hence, the measured torque is proportional to the lift and although the results are presented in terms of torque, the variations of Q will be treated as though they were variations of lift. We are primarily interested, of course, in the latter.

Upon inspection of Figure 8, it can be seen that the oscillatory motion of the airfoil gives rise to lifts which are not equal to those of the stationary airfoil at the same angles of attack. The lifts for all angular velocities considered are less than the corresponding static values. The reductions from the static values are least for high values of ω' during decrease of θ'; intermediate for $\omega' = 0$ (which condition is found only at the extreme position during an oscillation and is classified as "θ' increasing" as this motion has preceded the attainment of $\omega' = 0$); and greatest for high values of ω' with θ' increasing. The data points seem to define practically straight lines which pass through the origin.

The results will probably be, in one way, somewhat surprising to the reader. If intuition were relied upon, one would probably expect that lifts during increase of the angle of attack to be less than those corresponding to zero angular velocity, with a reversed deviation in the case of decreasing angles of attack. However, the reason for the differences between the lifts of the stationary airfoil and those obtained at $\omega' = 0$ during oscillation is not at once apparent. That this difference might have been expected will be shown below.

The character of the oscillatory motion and, more particularly, of the vortex system created by the motion, is so complex that a quantitative theoretical treatment of the problem would involve great mathematical difficulty. It has been found possible, however, to deduce purely qualitative answers to the questions of primary interest and the writer wishes to acknowledge his indebtedness to Mr. E. N. Jacobs, of the Langley Laboratory staff, for many valuable suggestions made during the development of the theoretical analysis which follows.

It will clarify the problem to consider the airfoil as rotating about an axis within itself, rather than about one at a considerable distance upstream, and this is justifiable. The compounding of velocities and derivation of the values of θ' and ω' transform, in effect, the actual motion to one of simple rotation about an axis at one-third chord aft of the leading edge of the airfoil.

It will be assumed that the circulation around the wing (1) during oscillation is negligibly different from that for the stationary wing at the same values of θ'. This infers, of course, that the Kutta hypothesis of smooth flow at the trailing edge is not materially violated. It will also be assumed that the vertical velocity induced by the entire vortex system is zero (2) at the wing when $\theta' = 0$. The first assumption is considered to be logical in view of the relatively small angular velocities attained in the tests. The second assumption is based upon the results of the tests and seems to be reasonably well borne out by approximate calculations for this particular case. More will be said later of the validity of these assumptions.

Let us take $\theta' = 0$, with the wing moving toward $+\theta'$, as a starting point. According to our assumptions the instantaneous lift is now zero. Let us consider the vortex system behind the wing to see how well our assumptions apply. The angle of attack is changing although $\theta' = 0$, that is, ω' is finite. According to Prandtl's explanation of the creation and variation of lift (Reference 2), transverse vortices are being discharged from the trailing edge of the wing and, because of the sense of rotation of the wing (toward $+\theta'$), their circulation will have a sense opposite to that of the circulation which would take place around the wing with positive lift. At the wing, the vertical velocity induced by these vortices is downward. Previous to the instant under consideration, the lift has been negative.

Therefore the vortices which trail aft from the tips of the airfoil have zero strength at the time when $\theta' = 0$ but are of finite and progressively greater strength as the distance behind the wing increases. The vertical velocity induced by these vortices is upward at the wing. Our second assumption simply amounts to the statement that the two components of induced vertical velocity must neutralize each other when $\theta' = 0$. (It will be shown later that this condition depends upon the aspect ratio.) With regard to the first assumption, the circulation around the wing must change sign as the lift does and the point of this reversal is put at $\theta' = 0$ as a first approximation.

Now let us turn our attention to the conditions existing a short time later, say when $\theta' = A$. The wing is at a positive angle of attack, the trailing vortices have grown to finite strength at the wing tips and the strength of the transverse vortices discharged per unit time has become slightly less than it was when $\theta' = 0$. Since, when $\theta' = 0$, the lift was zero, we have only to consider the effects of the vortices generated between the instants when $\theta' = 0$ and $\theta' = A$ to ascertain the order of the lift at the latter condition. (By this treatment we ignore the effects of all but the vortex elements which are close to the wing.) During the interval under consideration there have been discharged transverse vortices of total strength equal to Γ_A, the circulation around the wing at $\theta' = A$. These transverse vortices, with the trailing vortices of strength varying from zero at the position occupied by the wing when $\theta' = 0$ to Γ_A at $\theta' = A$, form a system of rectangular closed vortices as shown in Figure 9. A comparison of the vertical velocities induced by this "ladder" vortex system (Figure 10a) with those which would exist if the wing were stationary at $\theta' = A$ (corresponding vortex pattern shown as Figure 10b) will demonstrate how the induced angle of attack and the lift are affected. A simple calculation shows that the downward velocity induced at the "lifting line" LL, (Fig. 10), is greater in a than in b. It is concluded then, that the induced angle of attack is greater when θ' is increasing than when the wing is stationary at the same position. Therefore, the lifts attained while the angle of attack is increasing are less than the corresponding values for the stationary condition. This proof is applicable throughout the increase of the angle of attack.

As the angle of attack begins to decrease, the discharged transverse vortices undergo a reversal of sense; i.e., during this phase their circulation is of the same sense as that about the wing, and they induce upward velocities which tend to neutralize the downward components induced by the trailing vortices. The trend of the resultant induced vertical velocity is, of course, toward zero as θ' approaches zero again. While the angle of attack was increasing, both the longitudinal and transverse vortices contributed downward velocities at the wing;

now, however, we find the components of opposite sign. The result is that the induced angles of attack while θ' is decreasing, are less, and the lifts consequently greater, than those which were found while θ' increased. Although not demonstrated by the test results, it is not to be expected that the lift for the stationary condition is the limiting value for the lift with θ' decreasing; a sufficiently rapid decrease of θ' might lead to a much higher value.

It may now be interesting to investigate what the consequences would be if the second assumption were invalid. It is at once apparent that as the aspect ratio of the wing becomes greater, all other conditions remaining unchanged, the trailing vortices become incapable of neutralizing the effects of the transverse vortices at $\theta' = 0$. Consideration of the wing of infinite span will emphasize this point. Thus, in the case of a very long wing at $\theta' = 0$, with the angle of attack increasing, there will be a finite downwardly induced velocity at the wing. Although the geometric angle of attack θ' is zero, the effective angle of attack will be finite and negative. As the circulation depends upon the effective angle rather than the geometric angle, a negative circulation will exist and a negative lift will result. When we turn to the wing of very small span, the relative importance of the transverse vortices is lessened and the longitudinal ones assume control. It would seem probable then, that for wings of very small aspect ratios, it would be possible to create positive lifts at $\theta' = 0$ if θ' were increasing at a rapid rate.

To summarize the foregoing, it appears that, with all other conditions fixed, as the aspect ratio increases, the lift at $\theta' = 0$, with θ' increasing, will progress negatively; with θ' decreasing, the lift will progress positively. Obviously, at some finite aspect ratio, the lift at $\theta' = 0$ will be zero and it seems that the proportions of the wing used in these tests and the ratio of wing chord to the product of period of oscillation and velocity of translation were such that this balance condition was very nearly attained.

It is recommended that these tests be extended by the use of other aspect ratios and other periods of oscillation at the same air speed as that used herein. Tests in which undamped oscillations could be forced would also yield valuable results.

Conclusions

The results show that the instantaneous lift of a wing, while pitching, is not equal to the lift which would exist if the wing were stationary at the same angle of attack. The lift of the pitching airfoil is shown to be less than that of the stationary one although the reverse may be true if the lift is small or if the angle of attack is decreasing rapidly.

The action of the oscillating wing can be satisfactorily explained on the basis of accepted aerodynamic theory.

References

1. Munk, Max M. : Note on the Air Forces on a Wing Caused by Pitching. N.A.C.A. Technical Note No. 217 (1925).

2. Prandtl, L. : Applications of Modern Hydrodynamics to Aeronautics. N.A.C.A. Technical Report No. 116 (1921), p. 27.

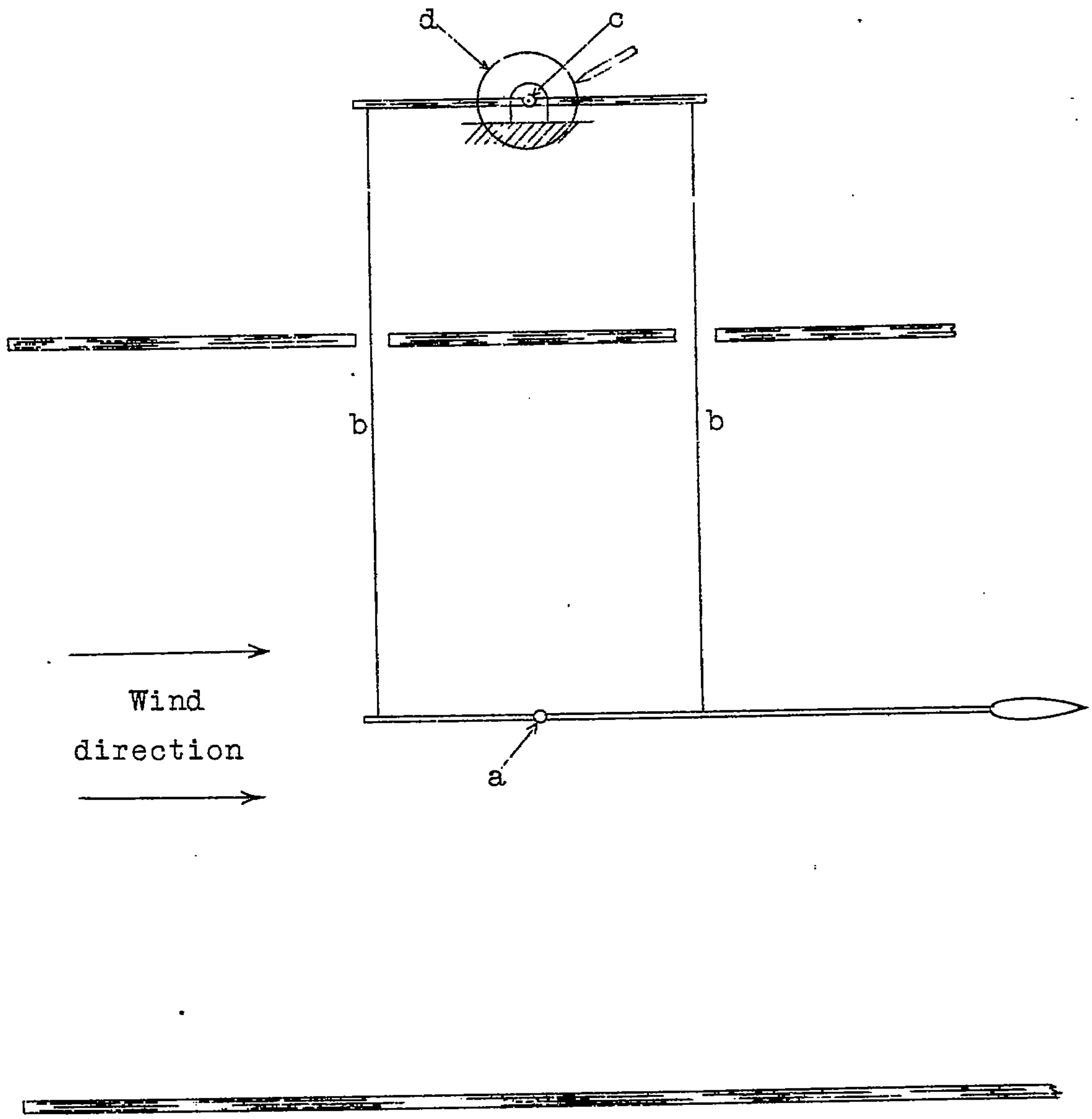


Fig.1.

Fig. 2 Oscillating airfoil in tunnel

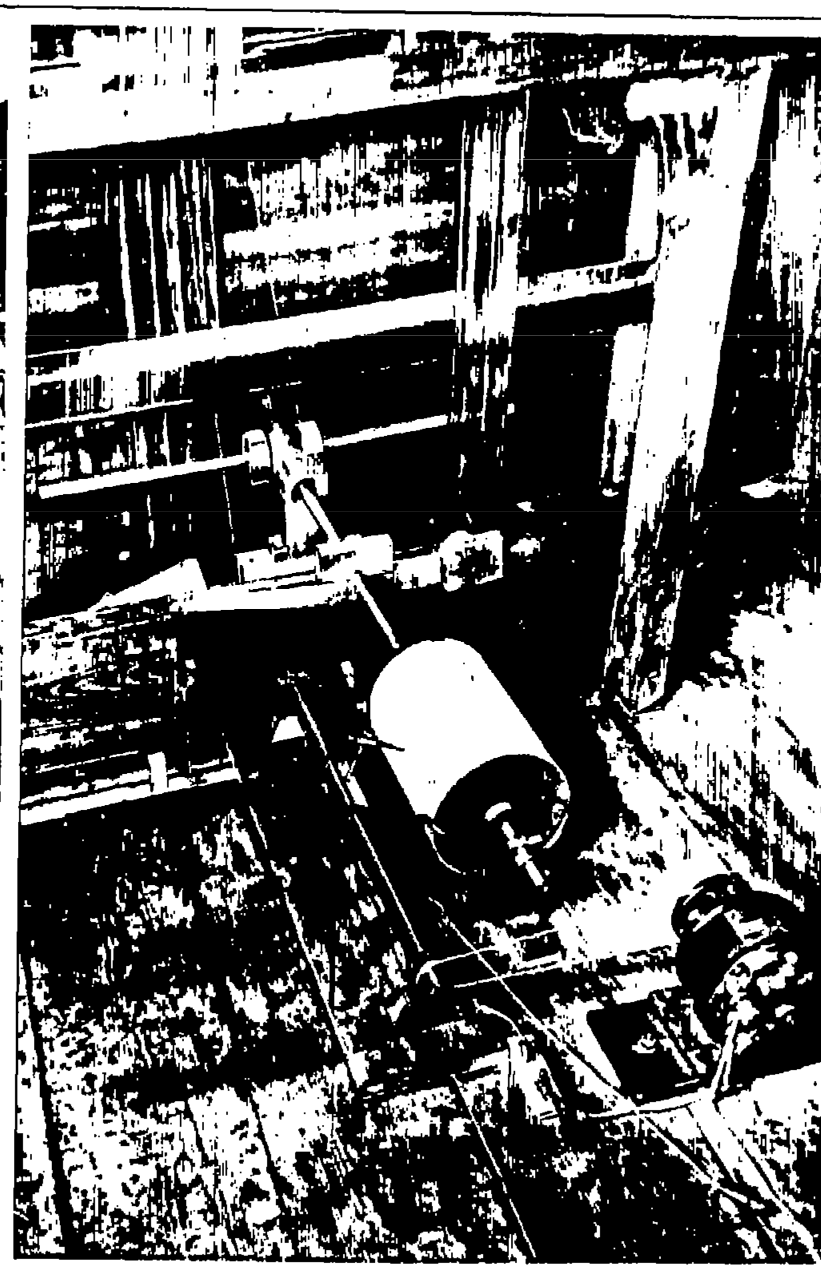
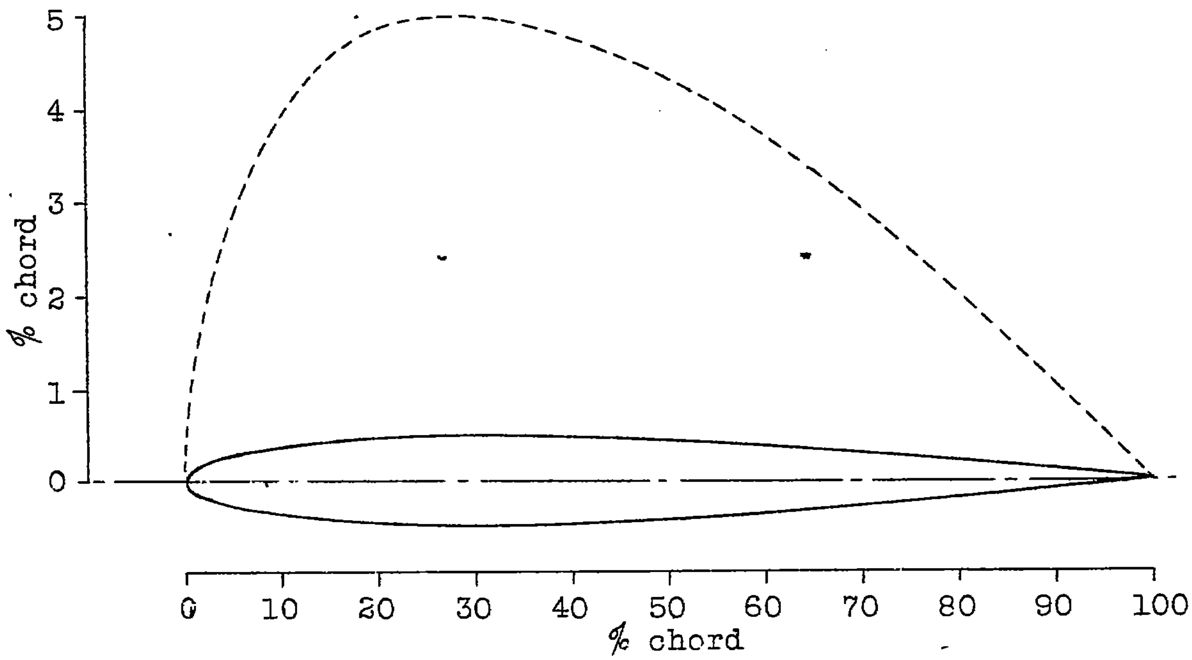


Fig. 3 Recording apparatus


5
4
3
2
1
0
% chord
0
10
20
30
40
50
60
70
80
90
100
% chord

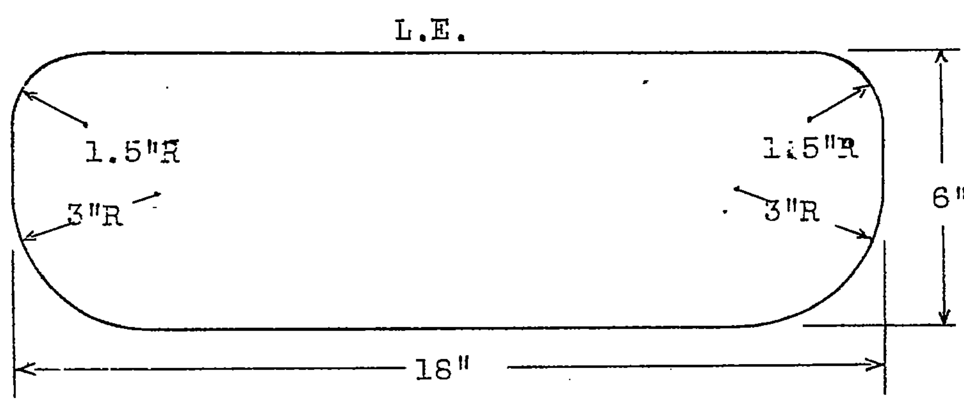

L.E.
1.5"R
3"R
1.5"R
3"R
6"
18"

Fig.4.

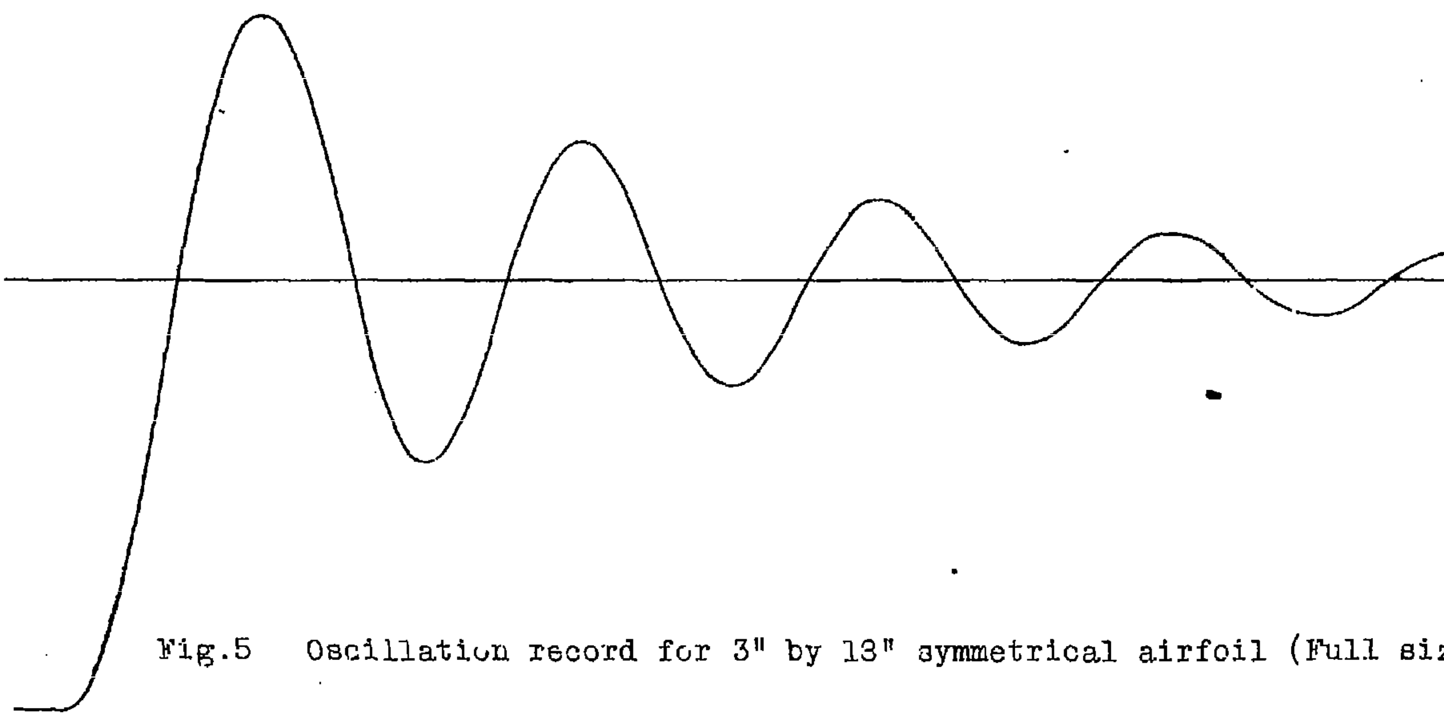


Fig.5 Oscillation record for 3" by 13" symmetrical airfoil (Full size)

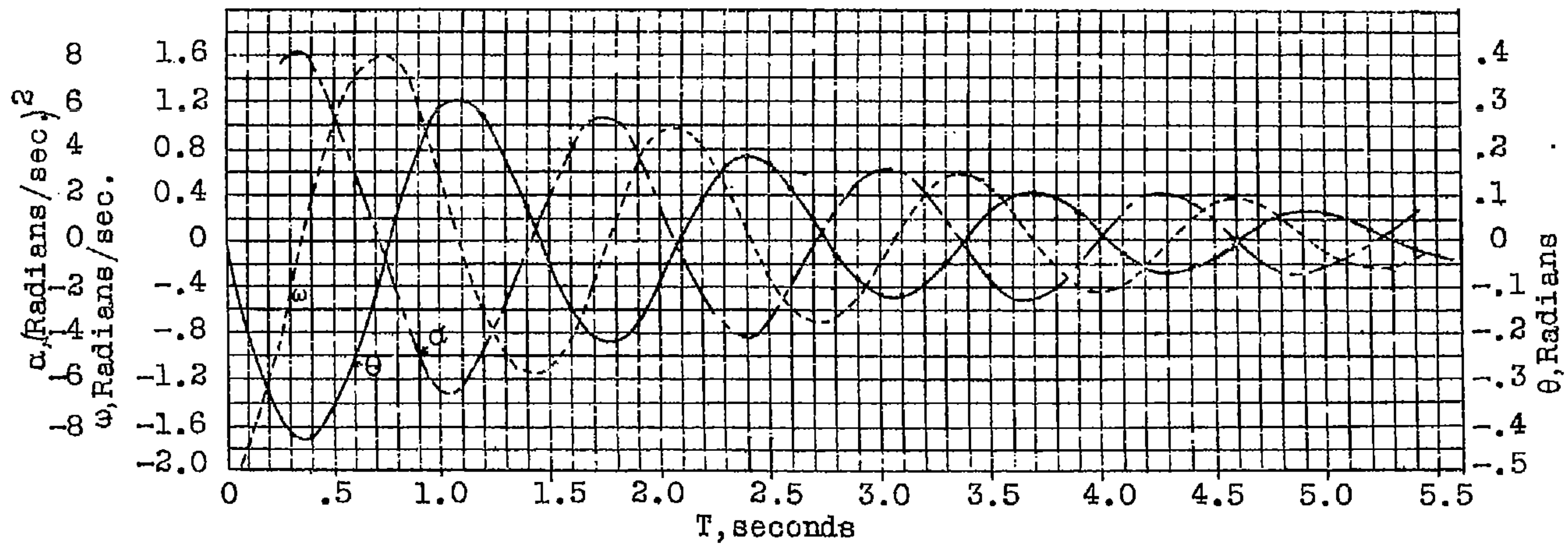

α,(Radians/sec.)^2
ω,Radians/sec.
θ,Radians
T,seconds
8
6
4
2
0
-2
-4
-6
-8
1.6
1.2
0.8
0.4
0
-.4
-.8
-1.2
-1.6
-2.0
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
ω
θ
α

Fig.6

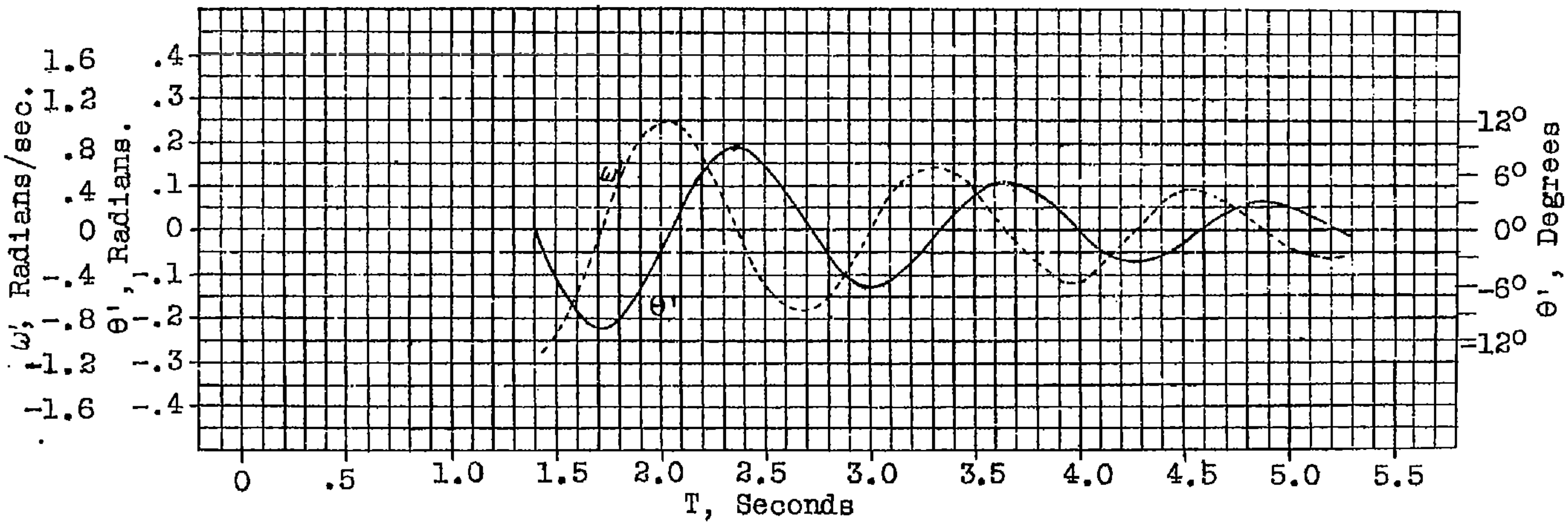

ω', Radians/sec.
1.6
1.2
.8
.4
0
-.4
-.8
-1.2
-1.6
θ', Radians.
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
ω'
θ'
12°
6°
0°
-6°
-12°
θ', Degrees
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
T, Seconds

Fig.7.

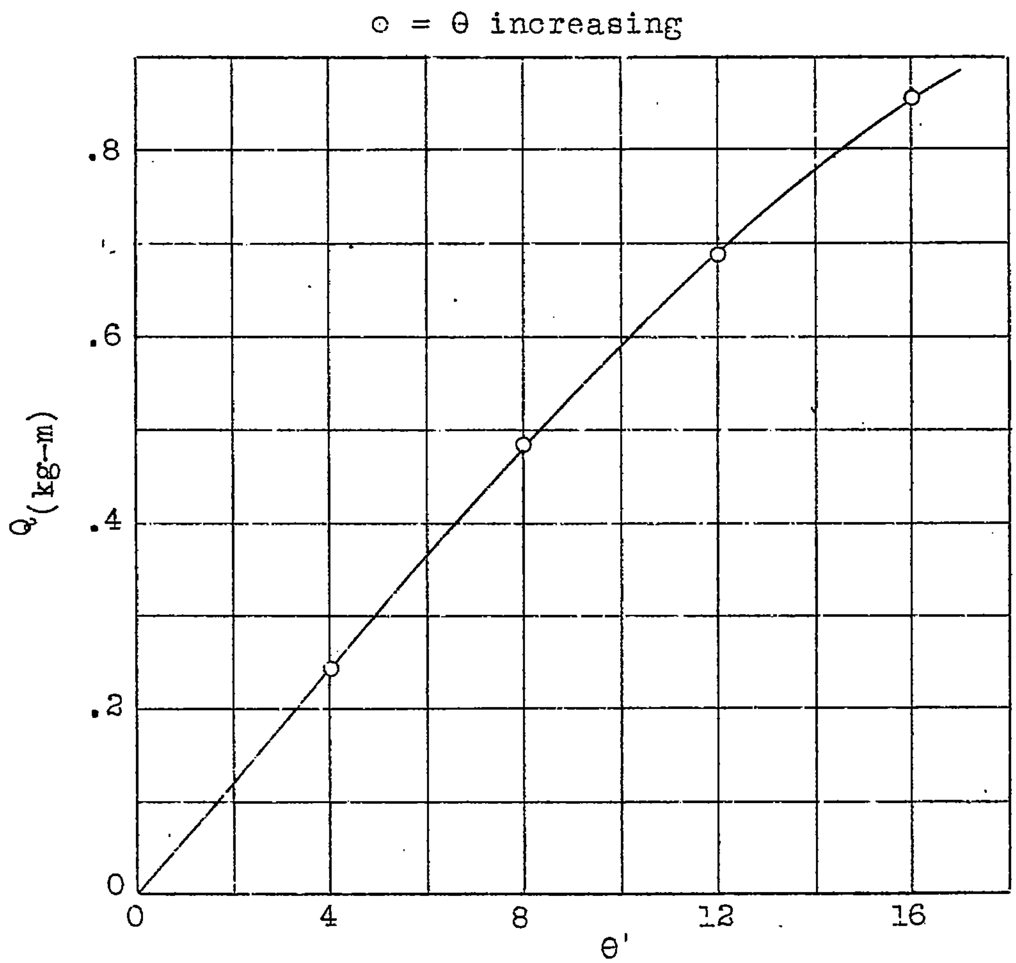


Fig.8a Stationary

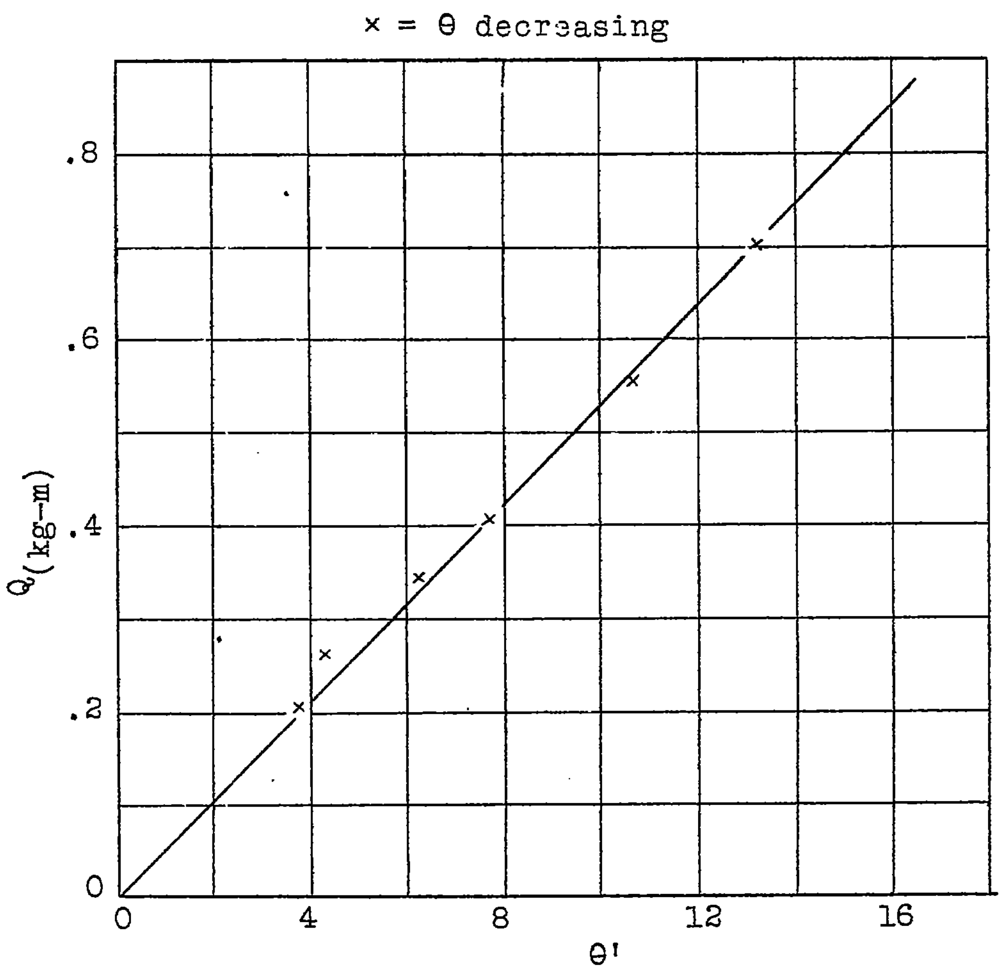


Fig.8b Oscillating, ω' = 0

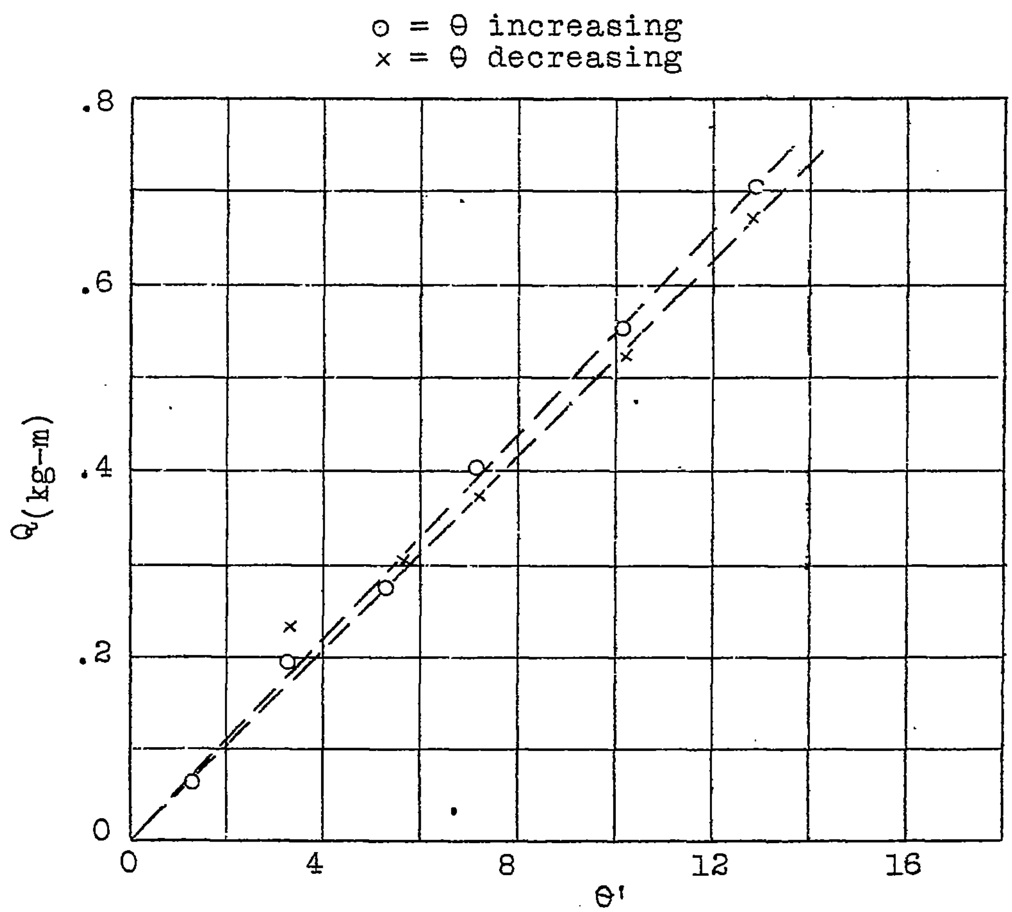


Fig.8c Oscillating, ω' = 0.25 rad./sec.

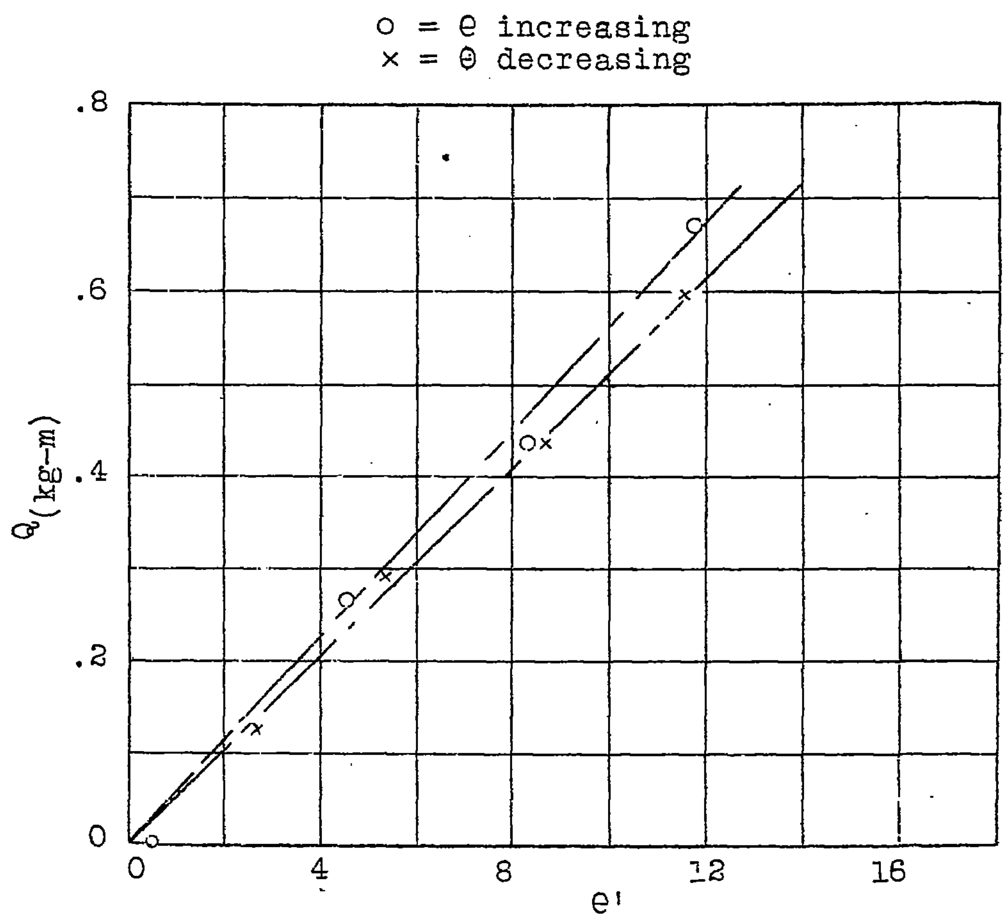


Fig.8d Oscillating, ω' = 0.5 rad./sec.

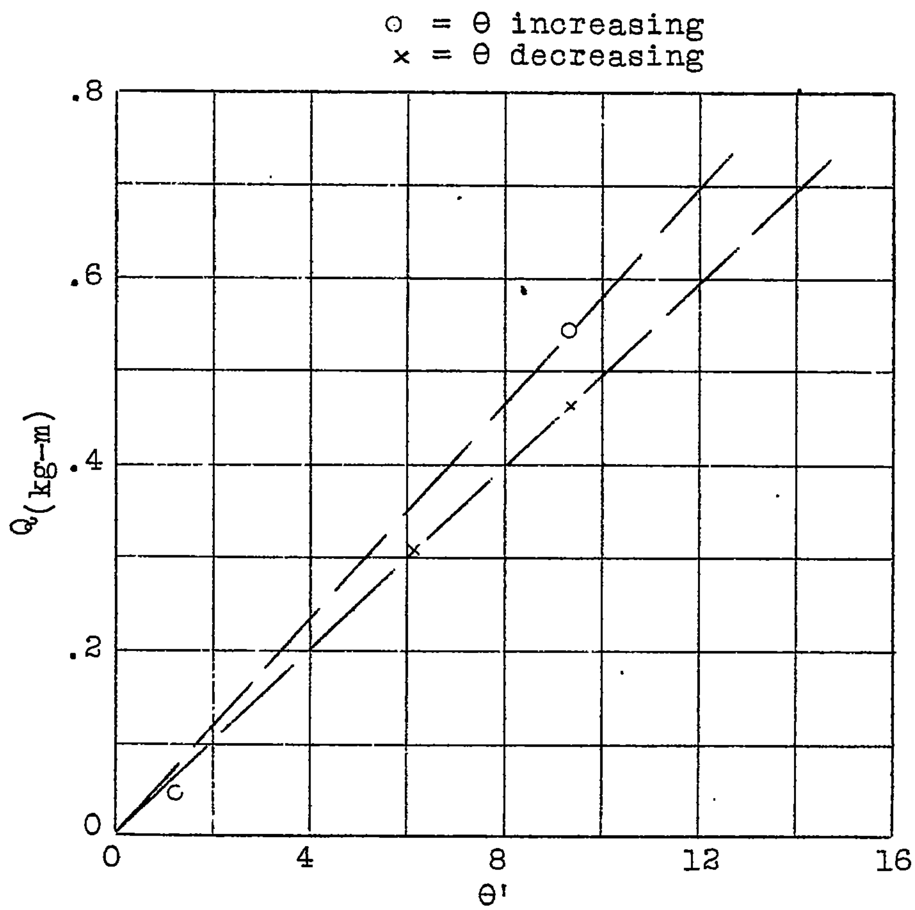


Fig.8e Oscillating, ω' = 0.75 rad./sec.

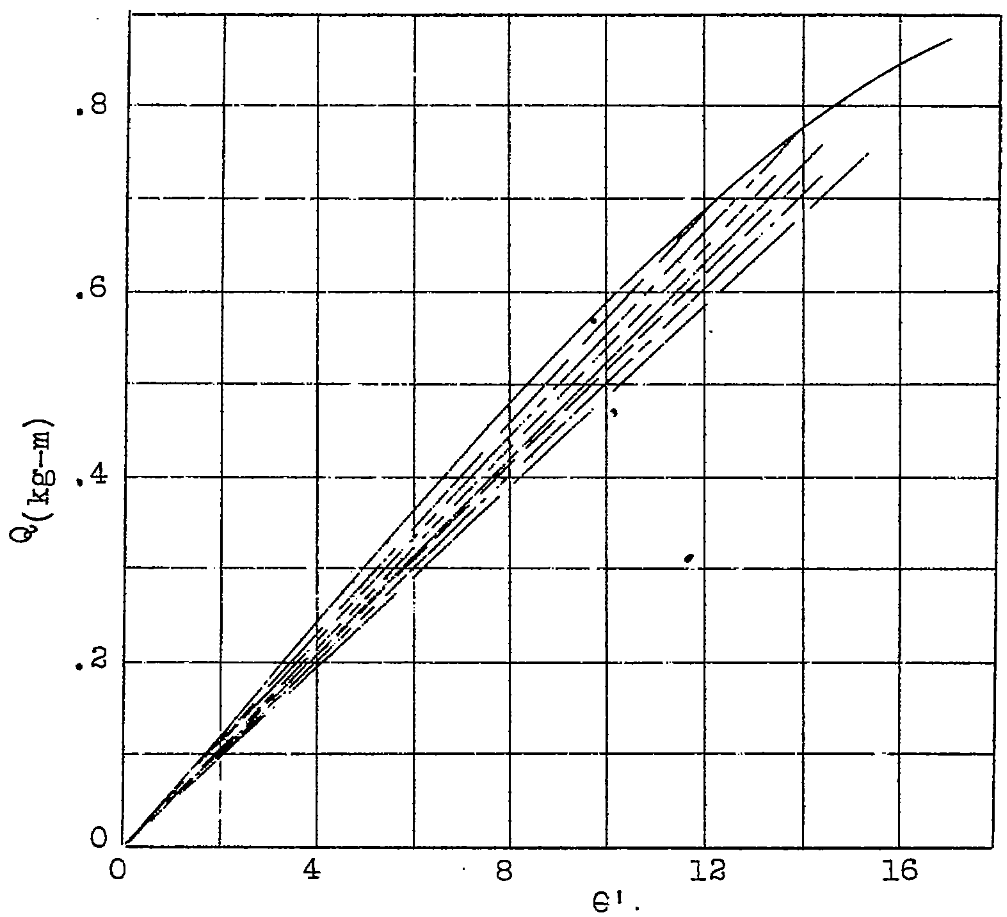


Fig.8f

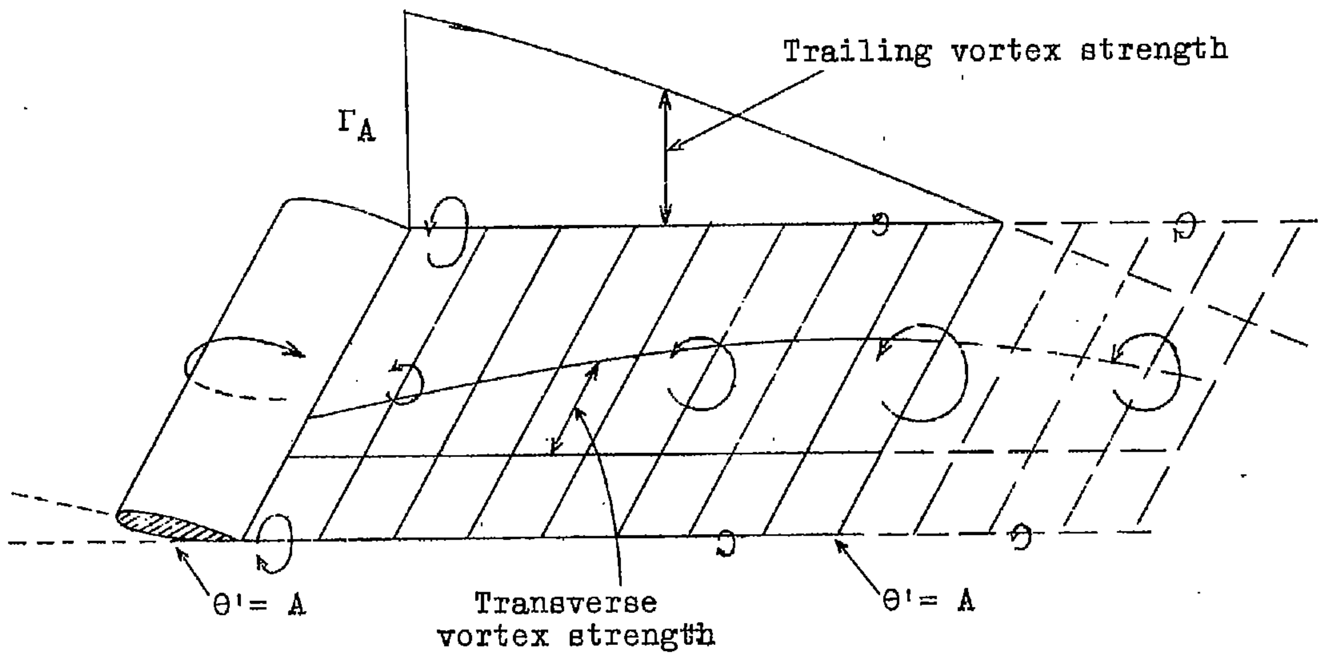
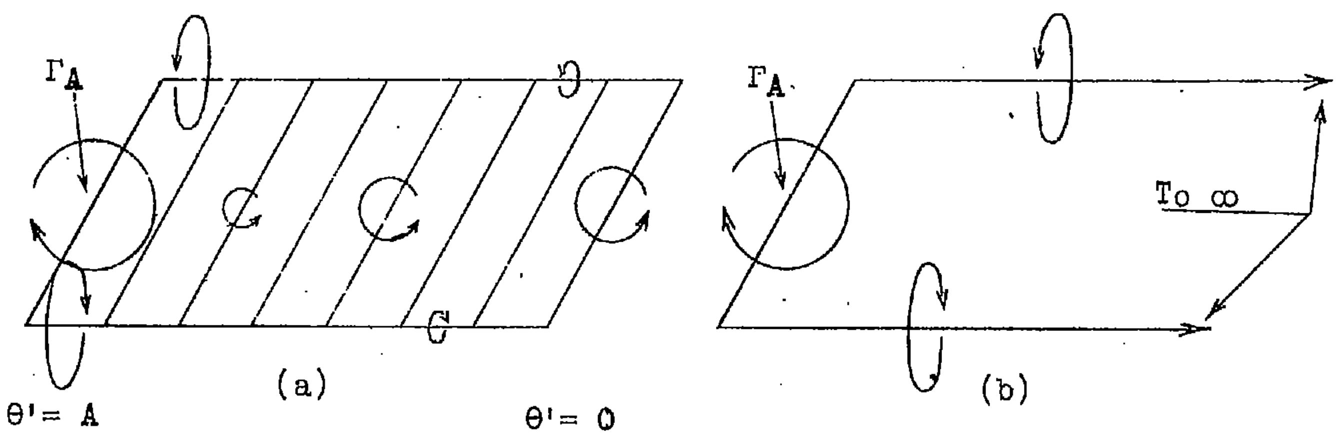

Trailing vortex strength
Γ_A
θ'= A
Transverse
vortex strength
θ'= A

Fig.9.



θ' Increasing

θ' Constant

Fig.10.